Examination of mean top height definitions and height estimation equations for *Pinus radiata* in New Zealand

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Abstract

For *Pinus radiata* in New Zealand mean top height is essentially defined and calculated via one definition and the so-called Petterson equation. A study shows that changing the definition to one used overseas has very little impact on top height values. The Petterson model appears to give unbiased estimates of tree height.

The basic relationship between tree height and diameter is not particularly precise, especially at low stockings. To ensure a satisfactory model, it is important that the sample trees have a wide range in diameter.

Changing the power parameter in the Petterson equation has a negligible effect. Adopting alternative height/diameter equations gives a minimal improvement in precision in one case but significantly less precision with the other.

Introduction

Woollons (2000) discussed a series of permanent sample plots established by the former forestry company NZ Forest Products, now a part of Carter Holt Harvey (CHH) Limited. They represented a subset of a database begun in 1933 that was later expanded to over 800 permanent sample plots. For many years the plot measurements were maintained and processed with a computer system built by company personnel at Tokoroa, but recently the records were moved to the Forest Research (FR) system at Rotorua. At the time of the transfer it was realised that former estimates of mean top height would likely change because of (a) two different definitions (given below) of the statistic and (b) different formula for estimating tree height when no direct measurement existed.

The CHH system is now defunct and it is purely academic to pursue a formal comparison of the two methods. However, it seems timely to review what is essentially the national system for estimating mean top height and ascertain whether more efficient or precise methods may be available. Mean top height should not be confused with predominant mean height (PMH), given by Goulding (1995):

The average height of the tallest tree, free of malformation, in each 0.01 ha plot within the stand.

In this contribution the effects of the two mean top height definitions and the FR estimation procedure are explored. The methodology is examined with data from an extensive thinning trial and the results are discussed.

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Background and Definitions

In even aged stands, the basic relationship between tree height and diameter (for a given age) is sigmoid in shape (Curtis 1967) although in practice a monomolecular form very usually suffices (Carron 1968) because measurements are normally not collected prior to the point of inflexion. For radiata pine the degree of curvature is not acute and sometimes nearly approaches a straight line especially at lower stockings. Fig. 1 shows the relationship at four ages, the data coming from an intensive *Pinus radiata* thinning trial discussed by Whyte and Woollons (1990).

Figure 1 Relationship between tree height and diameter at four ages.
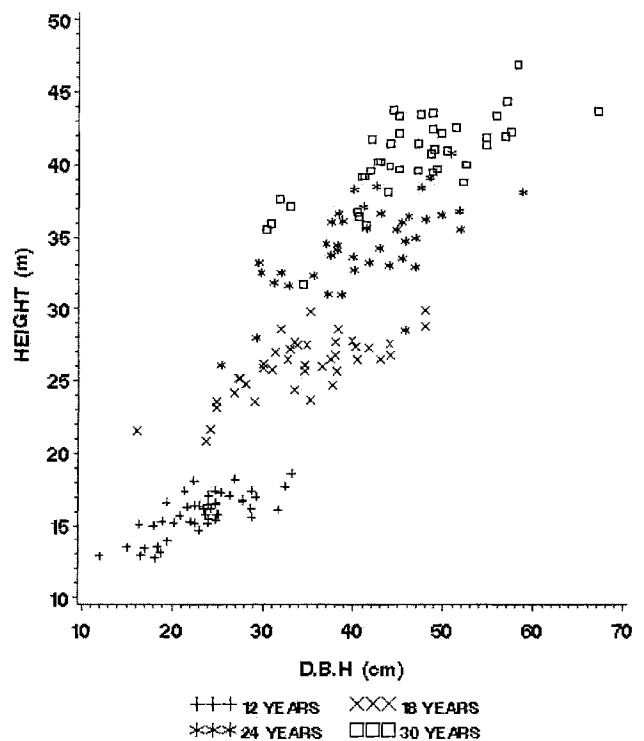
The definition of mean top height used by FR is that adopted by the New Zealand Institute of Forestry, and given by Goulding (1995):

The height predicted by the Petterson height/dbh curve for a dbh corresponding to the quadratic mean dbh of the 100 largest trees per hectare (based on dbh) in a stand.

A Petterson (1955) equation is given by:

$$H = 1.4 + (\beta + \alpha/D)^{-2.5} \qquad (1)$$

where H = tree height (m)

D = diameter at breast height (cm)

β, α = regression coefficients

For a given plot and for a particular measurement only that data (the height : diameter pairs) are used in the estimation of the coefficients in (1).

Equation (1) can be readily manipulated to a straight-line linear form

$$Y = D/(H - 1.4)^{0.4} = (\alpha + \beta D) \qquad (2)$$

where α and β can now be estimated by simple least squares formulae (for example, Draper and Smith 1998, p. 24-25). It is partially this property that makes (1) so useful especially in a historic sense when computing capability was limited.

An alternative definition of mean top height and that formerly used by CHH is that used in Great Britain (Johnston & Bradley 1963):

Mean top height is the average height of the 100 largest trees per hectare (based on diameter).

From the two definitions there are two ways of estimating mean top height. For a specific sample plot, the applicable top diameter trees need to be isolated; for example, if a sample plot occupies 0.1 hectare, then the biggest 10 trees by diameter represent the sample top diameter trees. Next, the height : diameter data from the sample plot are used to form a predictive equation of tree height, using model (1). We now have two options to estimate top-height.

(a) FR definition. Estimate top height as the predicted height given by equation (1) for a diameter corresponding to the quadratic mean of the top diameter trees.

(b) CHH definition. For the top diameter trees that do not have measured heights, predict them through (1) and estimate mean top height by the average of the actual or predicted tree heights.

The crucial difference lies in the averaging process. For FR, the diameters are averaged and one height is predicted. For CHH, all the top diameter trees have assigned heights (measured or predicted) and these are averaged.

Methods and Results

The Petterson equation was examined empirically by applying it to part of the data from 'Experiment 695' a thinning experiment reported by Woollons and Whyte (1990). The trial contrasted seven residual stockings: 200, 300, 400, 500, 600, 700 and 1200 stems/ha in four randomised blocks. Measurements were available, either annual or biennial, from age 7 to 30, but this study evaluated mean top height only at ages 12, 18, 24 and 30. At each measurement time and for all stockings, between 6 and 13 height : diameter pairs were obtained in each of the twenty-eight 0.1 ha plots.

Equation (2) was fitted to the data (for each plot and age) by ordinary least squares and used to predict top height in terms of the two definitions. Given the limitations of a small number of residuals for each model, there were no clear signs of bias. Table 1 gives the average (over the four replications) differences in top height estimation calculated from the definitions and procedures above for the respective stockings and ages.

Table 1: Average differences (m) of top height according to CHH and FRI definitions.

Stocking	Age (years)			
	12	18	24	30
200	0.07	0.02	-0.01	-0.10
300	-0.02	0.08	0.11	0.01
400	-0.05	-0.07	0.01	-0.14
500	0.05	-0.07	-0.11	-0.07
600	-0.10	-0.05	-0.18	0.00
700	0.02	0.05	-0.07	-0.18
1200	-0.04	-0.05	-0.20	-0.23

An analysis of variance for these differences showed weakly significant (p < 0.06) effects for both stocking and age. There is some trend for the differences to become bigger with the higher stockings and older ages but in absolute terms the differences are trivial.

The R^2 values derived from (2) are grossly inflated since the diameter variable appears on both sides of the equation. The models were recalculated using (1) using non-linear least squares for which approximate (Ratkowsky 1990) R^2 values are available. Table 2 gives the respective average R^2 figures for each stocking and treatment, together with the corresponding average standard error of the mean statistic. An analysis of variance for the plot multiple correlation data showed very significant (p < 0.0001) stocking effects, but nothing for stand age or the interaction. The standard error figures are more difficult to interpret because they become inflated at later ages partly through the absolute size of the trees. Within ages, there are no significant differences.

Table 2: Average (over 4 replications) R^2 values and standard error statistics of the derived equations.

Stocking	Age (years)			
	12	18	24	30
200	0.29 (0.35)	0.22 (0.45)	0.31 (0.56)	0.41 (0.52)
300	0.43 (0.34)	0.59 (0.42)	0.39 (0.63)	0.64 (0.56)
400	0.58 (0.29)	0.60 (0.44)	0.46 (0.60)	0.61 (0.52)
500	0.49 (0.33)	0.54 (0.44)	0.54 (0.63)	0.64 (0.56)
600	0.68 (0.29)	0.70 (0.37)	0.72 (0.56)	0.54 (0.66)
700	0.52 (0.33)	0.52 (0.48)	0.61 (0.71)	0.67 (0.71)
1200	0.70 (0.30)	0.66 (0.41)	0.66 (0.73)	0.71 (0.73)

Table 3 lists the average parameter values from model (1) for each age and stocking. Analyses of variance showed a highly significant (p < 0.0001) age effect for the parameter β but the effect of stocking and the interaction is non-significant. No significant effects emerged for the parameter α.

Table 3: Average parameter values of the Petterson equation.

Stocking	Age (years)							
	12		18		24		30	
	β	α	β	α	β	α	β	α
200	0.289	1.50	0.257	0.93	0.223	1.40	0.210	1.15
300	0.278	1.58	0.233	1.64	0.219	1.24	0.202	1.35
400	0.292	1.17	0.244	1.09	0.218	1.14	0.203	1.28
500	0.294	1.07	0.249	0.90	0.220	1.08	0.203	1.18
600	0.281	1.31	0.242	1.15	0.217	1.09	0.211	0.91
700	0.287	1.14	0.250	0.85	0.216	1.16	0.200	1.41
1200	0.289	0.90	0.242	0.86	0.215	1.08	0.199	1.37

The original Petterson equation used an exponent of three; Schmidt (1967) reported using a power of 2.5 instead. How it came to be changed to 2.5 in New Zealand I have not been able to establish. The data were re-run using equation (1) but successively using an exponent of 1.5 to 4.5 in one-half increments (It would have been neater to estimate the power term by nonlinear least squares but the small samples made this virtually impossible). There was a slight trend for powers of 3.0 and 3.5 to give smaller error sum-of-squares but in absolute terms the differences were very small.

Two alternative candidate equations to the Petterson are the log-reciprocal (Schumacher 1939) model:

$$H = 1.4 + \exp(\alpha + \beta/D) \qquad (3)$$

that can be estimated by ordinary least squares through

$$\ln(H - 1.4) = \alpha + \beta/D \qquad (4)$$

and the so-called Petterson 2 formulation

$$H = 1.4 + (\beta + \alpha D)^{-2.5} \qquad (5)$$

that transposes to

$$D/(H - 1.4)^{0.4} = (\alpha + \beta/D) \qquad (6)$$

Model (3) and (5) were fitted to the data and compared to the performance of (1). The average residual mean squares for the three models are (a) Petterson: 2.84 (b) Petterson 2: 3.31 and (c) log – reciprocal: 2.81 respectively. Paired t – tests between (a) and (b) and (a) and (c) were strongly significant, (p <0.005 and 0.0001) respectively.

Discussion

It will be noticed that the relationship between tree height and diameter at breast height is not especially precise (Schreuder *et al.* 1993), dominant height being relatively unaffected by within stand differences in stocking (as opposed to diameter) but likely to be subject to genetic and micro-site variation. Moreover, the actual measurement of height on mature trees, *per se*, is open to some error (Avery & Burkhart 1994).

Goulding (1995) warned that a poor choice of height : diameter trees can produce an inverted curve with larger diameter trees having lesser-predicted heights. This occurred with two of the 128 models assayed here; both were at age 30 where the negative slopes were

close to zero so the degree of bias was not large. More generally however, a potential problem is that very suppressed trees beyond the range of the sample may be assigned vastly inflated height estimates. This can usually be avoided by obtaining a wide range of diameter trees in the sample.

It is reassuring that usage of either definition makes essentially no difference to the estimation of mean top height. Initially, I found this slightly surprising and had expected larger differences by virtue of utilising a quadratic mean instead of the arithmetic mean. For example if we consider the numbers 1 and 2 then the quadratic mean is 1.58 but the arithmetic mean is only 1.50. (The quadratic mean cannot be smaller than the arithmetic mean). This difference is illusionary. For a larger sample and where the range of the data is relatively small (as will be the case with top element diameters) then the two means are virtually the same.

The R^2 values in Table 2 confirm that the precision of height/diameter regressions are not high. Beside the reasons given above precision is also limited by the small sample sizes usually employed in these regressions. The model formerly used by CHH was:

$$\ln(H - 1.4) = \alpha + \beta/D + \gamma/\sqrt{T} + \delta/\sqrt{T}.D \qquad (7)$$

where in (7)

$$T = \text{stand age in years}$$

For a given plot and as measurements accumulated, pooled data could be progressively utilised to estimate the model parameters so that at later ages a sample size of over 100 was common. A drawback of (7) however, was that predictions at older ages are influenced and weighted by data sometimes decades younger so that unless (7) fitted the data very well a small element of bias could be present (Woollons, CHH data). From Table 2 it is evident that prediction of tree height at low stockings is especially prone to low precision. While physiological reasons might be advanced to explain this in part, for example, more wind exposure (Maclaren *et al.* 1995) it is more likely to be caused by the narrow range of diameters usually present with low stockings. Basic regression principles dictate that a good predictive model will be obtained if there is a wide range in predictor values (Draper & Smith 1998)

The parameter β in the Petterson equation represents an asymptote or an upper limit to growth. Because of the reciprocal form of (1) smaller values of the parameter represent a higher asymptote. Logically, (from Table 3) the effect of age is highly significant as lower β values are successively estimated. There is a suggestion that the 200 stems/ ha treatment has lower asymptotic values perhaps indicative of the lower top height development noted for low stockings by Woollons *et al.* (1994) and Maclaren *et al.* (1995).

The ability of the power parameter in the Petterson equation to assume a range of values and give virtually identical accuracy is explained by the structure of (1). The power term is strongly correlated to the other

two parameters, so that any small change in the former will invokes small differences to the latter without any significant loss in predictive power.

The results of adopting other height/diameter models are reassuring in that the current equation clearly behaves well. The so-called Petterson 2 equation performed conspicuously worse, occasionally giving error mean squares estimates double that of the Petterson. While the log-reciprocal equation achieved a lower average error mean square, in absolute terms it is very small (0.8 %). Moreover, adoption of this equation would raise problems of back-transformation of logarithms and possible biases in prediction (Baskerville 1972, Flewelling & Pienaar 1981).

Conclusions

The Petterson equation as used by Forest Research gives an adequate prediction of tree height. Every care should be given to ensure a wide range of diameters is included in the sample trees. There is no evidence that changing the power term or adopting an alternative regression model would give practically better results.

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5-Year Review of Registration

The following individuals have made an application under Article 41 (2) of the NZIF Articles of Association for the Five Year Review of Registration as a Registered Forestry Consultant.

Harold D'arcy Corbett   (Kerikeri)
Tony Smith   (Wellington)

The review process as approved by Council allows any Member of the Institute the right to object to the continuation of a Member's Registration.

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New Application for Registration

The following individual has made an application under Article 40 of the NZIF Articles of Association for Registration as a Registered Forestry Consultant.

Stuart Desmond Orme   (Masterton)

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